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INTENSITY CALCULATIONS USING A GEOMETRICAL MODEL OF SNELL'S LAW--ETC(U)

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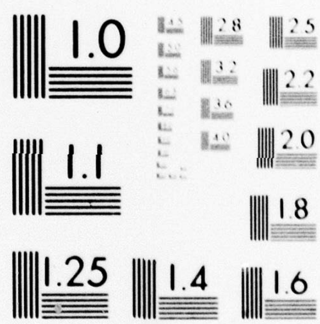
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(6) INTENSITY CALCULATIONS USING A GEOMETRICAL MODEL OF SNELL'S LAW,

by

(9) Technical memo, (10) Olgerts Puravs

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INTRODUCTION

R sub m

In USL Memorandum serial 1110-83 of 1 December 1953, by Mr. D. L. Cole, a geometrical model of Snell's Law was presented. It was further shown how the formula for the horizontal range increment, R_n, in a layer of water of a constant velocity gradient can be expressed in terms of velocities as prescribed by this model. The justification for this form of expression was argued to consist in the simplification of the programming of ray calculations. In a further memorandum (USL Confidential Technical Memorandum No. 1110-01-58) the geometrical model was also applied to the formula for incremental travel time, with the expectation of unifying the methods of calculation of both R_n and t_n solely in terms of the velocity structure.

R sub m   t sub m   I sub 0

It is the purpose of this report to extend these methods of computation and include the formula for the intensity ratio (I_0/I, where I_0 is the index intensity) at any point along the ray path in the above set. The following limitations should be noted:

(1) The formula for I_0/I here used does not take into account any intensity loss that may be incurred by attenuation. Spreading loss alone is considered.

(2) The spreading loss formula obtained here is invalid for calculations of intensity at all points on the limiting ray beyond the point where the ray reaches its vertex velocity.

TERMINOLOGY AND BASIC EQUATIONS

Before going any further the matter of terminology has to be somewhat better organized. In general the velocity of sound varies continuously

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with depth and this variation may not be linear. In refraction calculations it is therefore necessary to approximate the nonlinear variation of velocity with something linear, thus creating layers of fixed velocity gradients. In the discussion that follows, these layers will be indicated by the symbol n (n = 1,2,3,4,...) and numbered consecutively along the ray path in such a manner that the number of the layer increases by one each time the ray crosses a layer boundary or is reflected from it. For the purposes of simplified expression the point of origin of each ray, as well as the point of its termination, (the latter a matter of choice in each calculation) are considered as layer boundaries (see Fig. 1).

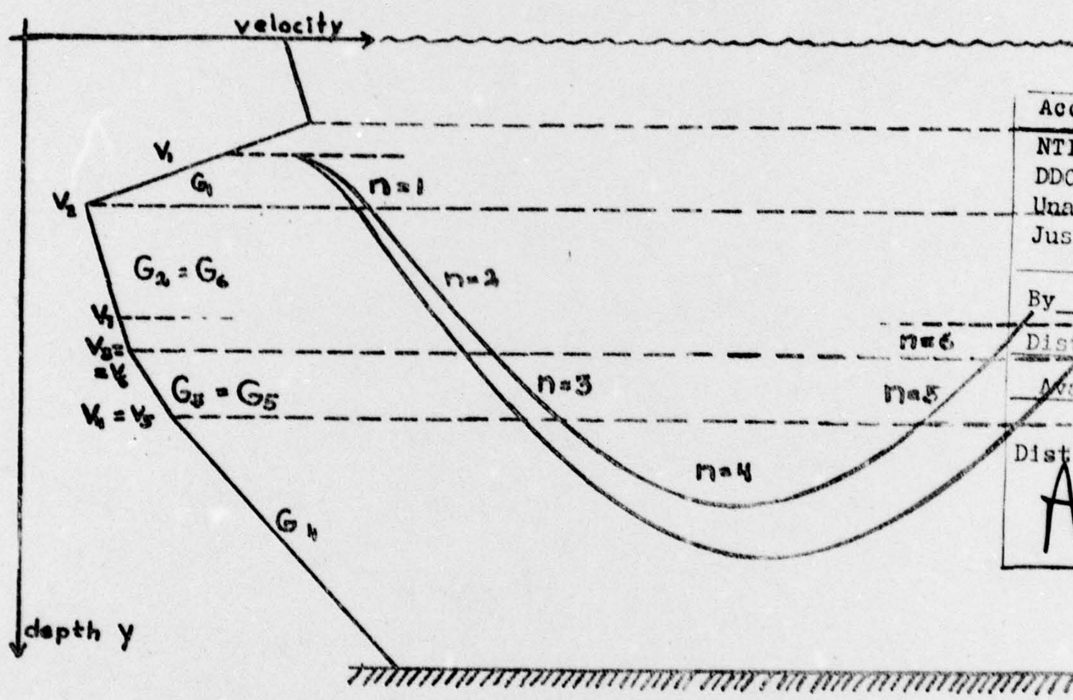

Figure 1

When n is attached as a subscript to some other symbol, it indicates that particular value of the symbol appropriate at the point where the sound ray crosses the boundary between layer (n-1) and layer n (see Fig. 1).

By way of completing our picture of the velocity structure of the ocean it is convenient to associate with each layer a constant velocity gradient:

(1) $$G_n = \frac{V_{n+1} - V_n}{y_{n+1} - y_n} \qquad (\sec^{-1})$$

where: V_n's - the velocity of sound at the points indicated (ft/sec)

y_n's - the depth at the same points (ft)

As a starting point the following equation will be used (for derivation see "Fundamentals of Sonar," J. W. Horton, U. S. Naval Institute, 1957, pages 99-100):

(2) $$\frac{I_0}{I} = \frac{R}{\cos\theta_1}\left|\sin\theta_{N+1}\left(-\frac{dR}{d\theta_1}\right)\right|$$

where: θ's - indicate the angles of inclination (with respect to the horizontal) of the ray (radians or degrees)

N - total number of layers passed through by ray

R - total horizontal range (yds)

The absolute value signs are utilized because we are interested in the separation between the bounding rays of a ray bundle and not in the direction in which this separation is measured. This equation is valid no matter what path the ray follows between its origin and the terminal points. To express it in terms of the geometric model we also need the relations between the trigonometric functions involved and the geometric model. The basic relation here is Snell's Law:

(3) $$V_x = \frac{V_1}{\cos\theta_1} = \frac{V_n}{\cos\theta_n} \qquad \left(\frac{ft}{sec}\right)$$

where V_x is the vertexing velocity of the sound ray. As the V_n's are all fixed by the velocity structure of the ocean, for a fixed source the only independent variable is θ_1. For each θ_1 a different ray path is specified because the ratio V_x is fixed by Snell's Law, thus making all θ_n's $(n \neq 1)$ dependent on the choice of θ_1. Assuming thus a fixed ray path with known V_x we have the following two relations:

(4a) $$\cos\theta_n = \frac{V_n}{V_x}$$

and from $\sin^2\theta = 1 - \cos^2\theta$

(4b)

$$\sin\theta_n = \frac{\pm\sqrt{V_x^2 - V_n^2}}{V_x}$$ (+ when θ_n is positive; - when θ_n is negative). This rule is reversed at each reflection from what it was before that reflection.

These relations will enable us to obtain the final expression for I_o/I entirely in terms of the velocity structure for each ray of fixed V_x .

Before this we have two other factors (R and $\frac{dR}{d\theta_1}$) involving the total range R which must be determined. In this determination we shall use two generalizations from the incremental range equations presented in USL Memorandum serial 1110-83. They are:

(5a) $$R = \sum_{n=1}^{N} R_n = \sum_{n=1}^{N} \frac{V_x}{3\,G_n}\left(\sin\theta_n - \sin\theta_{n+1}\right) =$$

(5b) $$= \sum_{n=1}^{N} \frac{1}{3\,G_n}\left[\left(\pm\sqrt{V_x^2 - V_n^2}\right) - \left(\pm\sqrt{V_x^2 - V_{n+1}^2}\right)\right] \text{(yds)}$$

where: R_n's - indicate the horizontal incremental ranges covered by sound ray in each constant gradient layer (yds).

The second form of the above equation (5b) is the result of direct substitution from (4b) and the sign convention of the latter is retained. It is important to remember that in ray calculations a ray directed downward forms a positive angle with the horizontal and the ray directed upwards a negative angle.

The derivative $dR/d\theta_1$ is calculated as shown in the appendix using (5a) and (3). Then, by means of (4a) and (4b) the derivative is expressed in terms of the velocity structure.

GENERAL CASE (N LAYERS)

The general expression is obtained now by substitution from Eqs. (4a), (4b), (5a) and (A7) into Eq. (2)

$$\frac{I_0}{I} = \frac{V_x \sum_1^N R_n}{V_1} \left| \frac{\pm\sqrt{V_x^2 - V_{N+1}^2}}{V_x} \; \frac{V_x^2\left(\pm\sqrt{V_x^2 - V_1^2}\right)}{V_1} \sum_1^N \frac{R_n}{\left(\pm\sqrt{V_x^2 - V_n^2}\right)\left(\pm\sqrt{V_x^2 - V_{n+1}^2}\right)} \right|$$

Since the velocities are positive and the two square roots may be always regarded as positive we can write the final form as follows:

$$(6) \quad \frac{I_0}{I} = \frac{V_x^2}{V_1^2} \sqrt{V_x^2 - V_1^2} \sqrt{V_x^2 - V_{N+1}^2} \sum_{n=1}^{N} R_n \left| \sum_{n=1}^{N} \frac{R_n}{\left(\pm\sqrt{V_x^2 - V_n^2}\right)\left(\pm\sqrt{V_x^2 - V_{n+1}^2}\right)} \right|$$

As stated before this equation is applicable in all cases except the two listed at the beginning. In all cases confusion will be avoided if Eq. (6) is reduced to its simplest form for each specific case before any numerical calculations are made.

A special case, sometimes giving rise to confusion, concerns the validity of the equation after a ray has passed its vertexing point and is crossing the layers in the opposite direction to that preceding the vertexing point. In this case the sign rules should be used as usual, taking into consideration any reflections that may have occurred, where they apply in Eq. (6). The choice of layers and layer boundaries should also be made in accordance with the directions previously given. Specifically, this means that the point of vertexing is not considered as a layer boundary and that the velocity $V_n = V_{n+1}$ (n designating vertexing layer) but the exit and entrance angles are opposite in sign. The range increment too in this case would be the distance between the two points at which the ray crosses the same boundary in opposite directions (see Fig. 1).

O. Puravs

O. PURAVS
Physicist

APPENDIX

Determination of $dR/d\theta_1$

Starting with Eq. (5a) from the report:

$$R = \sum_{n=1}^{N} \frac{V_x}{3 G_n} \left(\sin\theta_n - \sin\theta_{n+1} \right) \qquad \text{(yds)}$$

and expressing the dependent variable V_x in terms of the independent variable θ_1 by Snell's Law (Eq. (3)) we have

(A1)
$$R = \sum_{n=1}^{N} \frac{V_1}{\cos\theta_1} \frac{(\sin\theta_n - \sin\theta_{n+1})}{3\, G_n} \qquad \text{(yds)}$$

In Eq. (A1)both θ_n and θ_{n+1} are still dependent on θ_1 and this must be taken account of when differentiating. This operation is simplified, however, by avoiding the immediate replacement of these terms by functions involving θ_1 . Performing the differentiation we obtain:

(A2)
$$\frac{dR}{d\theta_1} = \frac{d}{d\theta_1}\sum_{n=1}^{N} R_n = \sum_{n=1}^{N} \frac{dR_n}{d\theta_1} = \sum_{n=1}^{N}\left[\frac{V_1}{3 G_n} \frac{\sin\theta_1}{\cos^2\theta_1}(\sin\theta_n - \sin\theta_{n+1}) + \right.$$

$$\left. + \frac{V_1}{3 G_n} \frac{1}{\cos\theta_1}\left(\cos\theta_n \frac{d\theta_n}{d\theta_1} - \cos\theta_{n+1}\frac{d\theta_{n+1}}{d\theta_1} \right)\right]$$

At this point we must complete the differentiation by obtaining the two derivatives $d\theta_n/d\theta_1$ and $d\theta_{n+1}/d\theta_1$ (for any value of n) from Snell's Law (Eq. (3)). Expressing Snell's Law as follows:

$$\cos\theta_n = \frac{V_n}{V_1} \cos\theta_1$$

and differentiating with respect to θ_1 we get:

$$-\sin\theta_n \frac{d\theta_n}{d\theta_1} = -\frac{V_n}{V_1}\sin\theta_1$$

or

(A3) $$\frac{d\theta_n}{d\theta_1} = \frac{V_n}{V_1}\frac{\sin\theta_1}{\sin\theta_n} = \frac{\cos\theta_n}{\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_n}$$

substituting back for V_n/V_1 from Snell's Law. Similarly:

(A4) $$\frac{d\theta_{n+1}}{d\theta_1} = \frac{\cos\theta_{n+1}}{\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_{n+1}}$$

Substitution of the Eq. (A3) and (A4) into Eq. (A2) allows for a considerable simplification as follows:

$$\frac{dR}{d\theta_1} = \sum_{n=1}^{N}\left[\frac{V_1}{3G_n}\frac{\sin\theta_1}{\cos^2\theta_1}\left(\sin\theta_n - \sin\theta_{n+1}\right) + \frac{V_1}{3G_n}\frac{1}{\cos\theta_1}\left(\frac{\cos^2\theta_n}{\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_n} - \frac{\cos^2\theta_{n+1}}{\cos\theta_1}\frac{\sin\theta_1}{\sin\theta_{n+1}}\right)\right] =$$

$$= \sum_{n=1}^{N}\frac{V_1}{3G_n}\frac{\sin\theta_1}{\cos^2\theta_1}\left[\left(\sin\theta_n - \sin\theta_{n+1}\right) + \left(\frac{\cos^2\theta_n}{\sin\theta_n} - \frac{\cos^2\theta_{n+1}}{\sin\theta_{n+1}}\right)\right] =$$

$$= \sum_{n=1}^{N}\frac{V_1}{3G_n}\frac{\sin\theta_1}{\cos^2\theta_1}\left(\frac{\sin^2\theta_n + \cos^2\theta_n}{\sin\theta_n} - \frac{\sin^2\theta_{n+1} + \cos^2\theta_{n+1}}{\sin\theta_{n+1}}\right)$$

(A5) $$\frac{dR}{d\theta_1} = \sum_{n=1}^{N}\frac{V_1}{3G_n}\frac{\sin\theta_1}{\cos^2\theta_1}\left(\frac{1}{\sin\theta_n} - \frac{1}{\sin\theta_{n+1}}\right)$$

This result may be further simplified by expressing it in terms of R_n. To achieve this the term in parenthesis should be put over a common denominator and Eq. (3) used to introduce V_x. Then by reversing signs and using Eq. (5a) we can substitute R_n into Eq. (A5).

$$\frac{dR}{d\theta_1} = \sum_{n=1}^{N} \frac{\sin\theta_1}{\cos\theta_1} \frac{1}{3G_n} \frac{V_1}{\cos\theta_1} \left(\frac{\sin\theta_{n+1} - \sin\theta_n}{\sin\theta_n \sin\theta_{n+1}} \right)$$

$$= \sum_{n=1}^{N} - \frac{\sin\theta_1}{\cos\theta_1} \left[\frac{V_x}{3G_n} \left(\sin\theta_n - \sin\theta_{n+1} \right) \right] \frac{1}{\sin\theta_n \sin\theta_{n+1}}$$

(A6) $$\frac{dR}{d\theta_1} = - \frac{\sin\theta_1}{\cos\theta_1} \sum_{n=1}^{N} \frac{R_n}{\sin\theta_n \sin\theta_{n+1}}$$

Substitution from Eq. (4a) and (4b) leads to the following final result:

(A7) $$\frac{dR}{d\theta_1} = \frac{-V_x^2 \left(\pm\sqrt{V_x^2 - V_1^2} \right)}{V_1} \sum_{n=1}^{N} \frac{R_n}{\left(\pm\sqrt{V_x^2 - V_n^2} \right)\left(\pm\sqrt{V_x^2 - V_{n+1}^2} \right)}$$

the signs being determined as explained in the report.

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